4. Positron Annihilation-in-Flight

The absolute intensity of 511-keV photons per 100 disintegrations ($\gamma^\pm(\%)$) from positrons annihilating at thermal energies in an absorber is:

$$
\gamma^\pm(\%) = 2[\beta^+(\%) - \beta_f^+(\%)],
$$

where $\beta^+(\%)$ and $\beta_f^+(\%)$ are the emitted and annihilated-in-flight absolute positron intensities, respectively.

There is a significant probability for annihilation-in-flight to result in two quantum annihilation (TQA) or one quantum annihilation (OQA) with a continuous photon energy distribution. The maximum photon energy is $E_0 + 1$, where $E_0$ is the maximum positron kinetic energy (endpoint) in units of the electron rest mass $m_e c^2$. The OQA probability for annihilation-in-flight of a positron of energy $E$ by collision with an atomic electron is given by Bethe\(^1\) as

$$
OQA \Phi(E, Z) = \frac{2\pi\alpha^4 Z^6 r_0^2 \left[ (E^2 + 2) + \frac{2}{3} E + \frac{4}{3} (E+2) \ln[E + (E^2 - 1)^{1/2}] (E^2 - 1)^{-1/2} \right]}{(E + 1)^2 (E^2 - 1)^{1/2}},
$$

and the TQA probability, as

$$
TQA \Phi(E) = \frac{\pi r_0^2 \left[ (E^2 + 4E + 1) \ln[E + (E^2 - 1)^{1/2} - (E + 3)(E^2 - 1)^{1/2}] \right]}{(E^2 - 1)(E + 1)},
$$

where $r_0 = 2.82 \times 10^{-13}$ cm is the classical electron radius, $\alpha = 1/137$ is the fine structure constant, and $Z$ is the atomic number of the absorber. Positron energies are given in units of the electron rest mass ($m_e c^2$).

The OQA probability given in equation (2) is generally small, except for high-Z absorbing materials where it is $\approx 16\%$ that of the TQA probability. Equation (2) includes collisions with electrons from the $K$ atomic shell only. The TQA probability given in equation (3) includes collisions with electrons from all atomic shells. The total probability for annihilation-in-flight by positrons of energy $E$ is given by\(^2\)

$$
P(E, Z) = \frac{N \rho}{A} \int_0^E [Z \Phi(E) + 2OQA \Phi(E, Z)](-dE/dx)^{-1}dE,
$$

where $N$ is Avogadro’s number, $A$, $\rho$ and $dE/dx$, are the atomic weight, the density, and the stopping power of the absorber, respectively.

Figure 7 shows the total probability for annihilation-in-flight of fully absorbed positrons in Be, Al, Cu, Ag, Pb and U, calculated with equation (4). The integration was done numerically and the probability corrected for the fact that a positron that has already annihilated can not reappear at a lower energy. The stopping power, $(dE/dx)$, was calculated for both collision- and bremsstrahlung-energy losses; for collisions, as described by Nelms,\(^3\) with mean excitation energies and density-effect corrections of Sternheimer\(^4\); for bremsstrahlung, as described by Koch and Motz.\(^5\)


Figure 7. Probability for annihilation-in-flight of fully absorbed positrons in various media.

- Be
- Al
- Cu
- Ag
- Pb
- U